Phase transitions in moving systems

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The general stability criteria of the supercritical Ginzburg-Landau equations in moving media are considered for different forms of the convective velocity which may change in space and time both periodically and randomly. The results are correlated with experiments on the propagation of vortices in superconducting films under the influence of a bias current. The role of the finite size of a sample is discussed.

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I. INTRODUCTION

A. The Ginzburg-Landau equation in moving systems

A well-accepted way of studying phase transitions is by means of the supercritical Ginzburg-Landau equation for the order parameter Ψ,

$$\frac{\partial \Psi}{\partial t} = \frac{\partial^2 \Psi}{\partial x^2} - \frac{\delta F}{\delta \Psi} = \frac{\partial^2 \Psi}{\partial x^2} + a\Psi - b\Psi^3, \tag{1}$$

where the homogeneous part of the free energy which is connected with a phase transition has the simplest analytical form $F = -a\Psi^2/2 + b\Psi^4/4$, and the homogeneous solutions $\Psi = 0$ and $\Psi = \sqrt{a/b}$ describe the disordered and ordered phases, respectively. For the case of superconductivity, the phenomenological equation (1) was derived afterward [1] from the microscopic theory of superconductivity with the function Ψ proportional to the local value of the energy gap parameter.

Usually one considers phase transitions in immobile systems, where the full derivative in time is replaced by the partial derivative, $d\Psi/dt \to \partial\Psi/\partial t$. However, there are some examples where the particles undergoing a phase transition are carried along by the mean flow that passes through the region under study. These include problems of phase transitions under shear [2], open flow of liquids [3], Rayleigh-Bénard and Taylor-Couette problems in fluid dynamics [4], dendritic growth [5], and chemical waves [6]. An additional example, which we considered earlier [7], is the motion of vortices in superconductors.

In all the above examples Eq. (1) will include the convective velocity v,

$$\frac{\partial \Psi}{\partial t} + v\frac{\partial \Psi}{\partial x} = \frac{\partial^2 \Psi}{\partial x^2} + a\Psi - b\Psi^3. \tag{2}$$

The magnetic field enters a type-II superconductor in the form of quantized objects—vortices. The motion of vortices is the subject of intensive study [8], as well as the related problems of dynamics of the interfaces in superconductors [9], and the effect of fluctuations on propagating fronts [10]. However, a new phenomenon, the formation of an ordered vortex phase in superconducting films subjected to the simultaneous action of both magnetic field and bias current, has only recently become a subject of experimental study [11]. Due to the nonhomogeneous surface potential barrier and a strong vortex interaction with spatial defects (pinning centers), the vortices penetrate a superconductor in a disordered vortex state. If the magnetic field is smaller than some temperature-dependent critical magnetic field B^*, an ordered phase will appear at certain distance from the sample edge. The temperature dependence in Eq. (1) is replaced now by that of the magnetic field, i.e., $a = a_0(B^* - B)$. The external magnetic field B is assumed to be constant ($B < B^*$) and with it the coefficient a in Eq. (2). The control parameter in this case is the bias current of density J. This current will drag the vortices along the sample, helping them to destroy the disordered phase by assisting the vortices to climb the pinning barriers. The transformation of the disordered vortex phase into an ordered one in the presence of a bias current can be directly observed by magneto-optical measurements of high temporal resolution, where a sharp interface between the ordered and disordered vortex phases has been detected [11].

In general, one can estimate the coefficients v and a in Eq. (2) in the following way. According to the Lorentz law, the velocity v is defined by the force acting on the vortex with flux quantum Φ_0,

$$v = \frac{F}{\eta} = \frac{J \times \Phi_0}{c\eta}, \tag{3}$$

where η is the friction coefficient of the vortices [12].

In order to estimate the coefficient a, assume that the coefficients a and b have the simplest analytical form

$$a(H) = a_0(B^* - B), \quad b(B) = b(B^*) = b_c. \tag{4}$$

Then the free energy difference ΔF near the boundary between ordered and disordered phases, $\Delta F = a^2/b$, is equal to the magnetic energy, that is,

$$\frac{H^2}{4\pi} = \frac{a^2}{b} = \frac{a_0^2(B^* - B)^2}{b_c}, \tag{5}$$

and the coefficient a_0 is defined by the magnetic susceptibility at the phase boundary.

Note that the experiments described in [11] were performed on $2.6 \times 0.3 \times 0.05$ mm^3 and $2.4 \times 0.3 \times 0.02$ mm^3 samples of $NbSe_2$ at temperature 5 K, magnetic field 0.4 T, and current density of the order of 1 mA mm^{-2}. In this case the vortex velocity v was of the order of 10^{-3} m sec^{-1}, in accordance with Eq. (3).

These experiments can be expanded to ac currents, to systems with periodically ordered pinning centers, etc. In the interpretation of these and similar experiments one has to take into account the presence of noise. It is just the aim of this work to prepare the basis for analyzing such experiments.

B. Solution of the linearized equation with constant velocity

For the stability analysis we will use the linearized version of Eq. (2):

$$\frac{\partial \Psi}{\partial t} + v\frac{\partial \Psi}{\partial x} = \frac{\partial^2 \Psi}{\partial x^2} + a\Psi. \tag{6}$$

One can eliminate the term in v by defining a new function $\Gamma(x,t)$ such that

$$\Psi(x,t) = \Gamma(x,t)\exp\left(\frac{vx}{2} - \frac{v^2 t}{4}\right). \tag{7}$$

On substituting Eq. (7) into Eq. (6), one gets

$$\frac{\partial \Gamma}{\partial t} = \frac{\partial^2 \Gamma}{\partial x^2} + a\Gamma, \tag{8}$$

i.e., the stability analysis of the linearized version of the Ginzburg-Landau equation (6) with constant convective velocity is similar to that of the appropriate equation (8) for immobile systems. The solution of Eq. (8) is proportional to

$$\exp\left[at - \frac{x^2}{4t}\right]. \tag{9}$$

Then, according to Eqs. (8) and (9), the exact solution of Eq. (6) will contain an exponential of the form

$$\exp\left[at - \frac{(x-vt)^2}{4t}\right]. \tag{10}$$

The exact solution of the linearized equation (6) can be found only for a constant convective velocity. When this velocity contains periodic or random terms we content ourselves with approximate solutions.

C. Convective and absolute instability

In contrast to immobile systems, there is more than one type of instability in moving systems. The growing mode can be shifted by flow so that locally a system remains stable, and the phase boundary is moving downstream (convective instability), while for an absolute instability the phase boundary is moving both downstream and upstream, eventually covering the entire system. The following simple arguments [13] illustrate these two possibilities.

The exact solution of the linearized equation (6) contains the exponential form (10) which describes the propagating wave packet. For each t one can find two values of x, $x = [(v \pm \sqrt{4a})t]$, which define the behavior of the two "edges" of the wave packet. For $a < 0$, there are no real x, i.e., no divergent rays, and the system is stable. For $v^2 > 4a$, both values of x have the same sign, and the solution $\Psi \neq 0$ of Eq. (6) is carried away with the convective velocity v (convective instability). Finally, for $v^2 < 4a$, the edges $x_{1,2}$ have different signs, i.e., the wave $\Psi \neq 0$ moves in both directions (absolute instability).

II. SOLUTION OF THE LINEARIZED EQUATION WITH VARYING VELOCITY

A. Slowly varying velocity

Let us consider our basic equation (6) in the case where the velocity is a slowly varying function of the spatial position x,

$$\frac{\partial \Psi}{\partial t} = \frac{\partial^2 \Psi}{\partial x^2} - v(\epsilon x)\frac{\partial \Psi}{\partial x} + a\Psi, \tag{11}$$

where $\epsilon \ll 1$. The full analysis of Eq. (11) with $a = a(\epsilon x)$ and $v = \text{const}$ has been performed by Hunt and Crighton [13]. By substituting

$$\Psi(x,t) = \Phi(x,t)\exp\left(\frac{1}{2}\int_0^x v(\epsilon x')dx'\right), \tag{12}$$

one gets the following equation for $\Phi(x,t)$:

$$\frac{\partial \Phi}{\partial t} = \frac{\partial^2 \Phi}{\partial x^2} + \left[a + \frac{1}{2}\frac{dv(\epsilon x)}{dx} - \frac{[v(\epsilon x)]^2}{4}\right]\Phi. \tag{13}$$

Thus, for a linear function $v(\epsilon x) = v_0 + v_1 \epsilon x$, one gets

$$\frac{\partial \Phi}{\partial t} = \frac{\partial^2 \Phi}{\partial x^2} + \left[a - \frac{v_0^2}{4} + \frac{1}{2}\epsilon v_1 - \frac{1}{2}v_0 v_1 \epsilon x - \frac{1}{4}v_1^2 \epsilon^2 x^2\right]\Phi. \tag{14}$$

Hence, with the help of Eq. (12) the linear variation of $v(\epsilon x)$ in Eq. (11) transforms into a quadratic variation of $a(\epsilon x)$ in Eq. (11) with $v = 0$. The analysis of this case [13] shows that the wave packet "edges" occur at

$$x^2 \sim \left\{\frac{4a_{\max}}{\epsilon v_1} - 2\right\}t, \quad a_{\max} \equiv \sqrt{a + \frac{\epsilon v_1}{2}}. \tag{15}$$

By analogy to the analysis performed above for $v = \text{const}$, one concludes that Eq. (15) has no real solutions for x, i.e., the system is stable, when, for $v_1 > 0$,

$$a_{\max} < \frac{\epsilon v_1}{2}. \tag{16}$$

This condition replaces the stability condition $a < 0$ for a constant convective velocity.

All the above analysis refers to the linear equation (6). However, the possibility exists that the nonlinearity in Eq. (2) dominates the inhomogeneity. This can happen, for example, when at the entrance $a(x=0)$ is already larger than $v^2/4$, i.e., the condition of absolute instability is satisfied, and the new mode develops at a distance $x = x_0$ such that the function $a = a_0 - a_1 x_0$ is still larger than $v^2/4$ [14].

B. Stability conditions for spatially dependent periodic damping

The motion of fluxons is interrupted by their captures by pinning centers. One can prepare a system in such a way that pinning centers are located periodically or quasiperiodically along the system [15]. Then the convective velocity will vary periodically along the system, and the stationary version of Eq. (6) takes the form

$$\frac{\partial^2\Psi}{\partial x^2} - v(1 + b\cos lx)\frac{\partial\Psi}{\partial x} + a\Psi = 0. \qquad (17)$$

Equation (17) with periodic coefficients has a Floquet solution of the form [16]

$$\Psi(x) = \exp(\alpha x)\psi(x) = \exp(\alpha x)\sum_{n=0}^{\infty} A_n \sin\left(\frac{nlx}{2}\right) + B_n\cos\left(\frac{nlx}{2}\right), \qquad (18)$$

where the periodic function $\psi(x)$ is expanded in a Fourier series. According to the Floquet theorem, the Floquet multiplier α must vanish at the stability boundaries. On substituting Eq. (18) with $\alpha=0$ into Eq. (17) and comparing the harmonics in front of the sine and cosine terms, one obtains an infinite system of linear equations for A_n and B_n which have nonzero solutions if the infinite determinant of these equations $\Delta(\alpha=0)$ vanishes, $\Delta(\alpha=0)=0$. One has to truncate this determinant at some n, and afterward to improve the result by taking into account the larger values of n. Leaving only terms with $n=1$, one obtains the following equations:

$$\left(a - \frac{l^2}{4} + \frac{lbv}{4}\right)A_1 + \frac{lv}{2}B_1 = 0,$$

$$-\frac{lv}{2}A_1 + \left(a - \frac{l^2}{4} - \frac{lbv}{4}\right)B_1 = 0. \qquad (19)$$

Equations (19) have nontrivial solutions if the determinant of these equations vanishes, which gives

$$b = \sqrt{4 + \frac{(4a-l^2)^2}{v^2l^2}}. \qquad (20)$$

The stability boundary (20) of the solution $\Psi=0$ has a V form in the b-l plane with the stable state located inside this curve.

Let us consider now a special case of Eq. (17), when the amplitude of the periodic force is small.

C. Slightly modulated convective velocity

One can find the approximate solution of the linearized Ginzburg-Landau equation with the modulated convective velocity

$$\frac{\partial\Psi}{\partial t} = \frac{\partial^2\Psi}{\partial x^2} - v(1 + b\cos lx)\frac{\partial\Psi}{\partial x} + a\Psi \qquad (21)$$

for the case of a weak modulation $b \ll 1$. As was done in Eqs. (7) and (8), one can eliminate the term in v by defining a new function $\Gamma(x,t)$ such that

$$\Psi(x,t) = \Gamma(x,t)\exp\left(\frac{vx}{2} - \frac{v^2t}{4}\right). \qquad (22)$$

On substituting Eq. (22) into Eq. (21), one gets

$$\frac{\partial\Gamma}{\partial t} = \frac{\partial^2\Gamma}{\partial x^2} - vb\left(\frac{\partial\Gamma}{\partial x} + \frac{v}{2}\Gamma\right)\cos lx + a\Gamma. \qquad (23)$$

Since b is assumed to be a small parameter, we will expand the function Γ in a perturbation series as

$$\Gamma(x,t) = \sum_{n=0}^{\infty}\Gamma_n(x,t)b^n, \qquad (24)$$

and consider only the two lowest-order terms in the expansion. These are readily seen to satisfy

$$\frac{\partial\Gamma_0}{\partial t} = \frac{\partial^2\Gamma_0}{\partial x^2} + a\Gamma_0,$$

$$\frac{\partial\Gamma_1}{\partial t} - \frac{\partial^2\Gamma_1}{\partial x^2} - a\Gamma_1 = -v\left(\frac{v}{2}\Gamma_0 + \frac{\partial\Gamma_0}{\partial x}\right)\cos lx. \qquad (25)$$

Equations (25) can be easily solved with the appropriate initial and boundary conditions.

D. Space-dependent random velocity

We consider a linearized equation (6) with the coordinate-dependent random velocity

$$\frac{d^2\Psi}{dx^2} - v[1 + \xi(x)]\frac{d\Psi}{dx} + a\Psi = 0, \qquad (26)$$

where the random force $\xi(x)$ is a Gaussian variable with zero mean and white noise correlator

$$\langle\xi(x)\xi(x_1)\rangle = D\delta(x - x_1). \qquad (27)$$

Let us rewrite Eq. (26) as

$$L\{\Psi\} = v\xi\frac{d\Psi}{dx}, \qquad (28)$$

where

$$L\{\Psi\} \equiv \left(\frac{d^2}{dx^2} - v\frac{d}{dx} + a\right)\Psi. \qquad (29)$$

In order to convert the differential equation (26) into an integro-differential equation we apply, following [17], the operator L^{-1} to Eq. (28), which gives

$$\Psi = L^{-1}\left\{v\xi\frac{d\Psi}{dx}\right\}. \qquad (30)$$

Using the result that $L[L^{-1}\{f\}] \equiv f$, one can easily check that the integral operator L^{-1}, which is inverse to the differential operator L defined in Eq. (29), has the following form:

$$L^{-1}\{f\} \equiv \frac{1}{a_1}\int_0^x dx_1 \exp\left[\frac{v}{2}(x - x_1)\right]\sin[a_1(x - x_1)]f(x_1),$$

$$a_1 = \sqrt{a - \frac{v^2}{4}}, \qquad (31)$$

i.e.,

$$\Psi(x) = \frac{v}{a_1}\int_0^x dx_1 \exp\left[\frac{v}{2}(x-x_1)\right]\sin[a_1(x-x_1)]\xi(t_1)\frac{d\Psi}{dx}(x_1) \tag{32}$$

and

$$\frac{d\Psi}{dx} = \frac{v}{a_1}\int_0^x dx_1 \exp\left[-\frac{v}{2}(x-x_1)\right]\xi(x_1)\frac{d\Psi}{dx}(x_1) \times\left\{a_1\cos[a_1(x-x_1)] + \frac{v}{2}\sin[a_1(x-x_1)]\right\}. \tag{33}$$

On substituting Eq. (33) into Eq. (28), one obtains

$$\left(\frac{d^2}{dx^2} - v\frac{d}{dx} + a\right)\Psi(x) = \frac{v^2}{a_1}\int_0^x dx_1 \exp\left[\frac{v}{2}(x-x_1)\right] \times\xi(x)\xi(x_1)\frac{d\Psi}{dx}(x_1) \times\left\{\frac{v}{2}\sin[a_1(x-x_1)] + a_1\cos[a_1(x-x_1)]\right\}. \tag{34}$$

On averaging of Eq. (34), for the noise defined in Eq. (27) one finds

$$\left\langle \xi(x)\xi(x_1)\frac{d\Psi}{dx}(x_1)\right\rangle = \langle\xi(x)\xi(x_1)\rangle\left\langle\frac{d\Psi}{dx}(x_1)\right\rangle = D\left\langle\frac{d\Psi}{dx}(x)\right\rangle. \tag{35}$$

The substitution of Eq. (35) into the averaging equation (34) shows that for white noise one gets

$$\left[\frac{d^2}{dx^2} - v(1+vD)\frac{d}{dx} + a\right]\langle\Psi\rangle = 0. \tag{36}$$

On comparing this equation with the stationary version of Eq. (6), one concludes that the existence of noise results in the renormalization of the velocity, $v \to v(1+vD)$, which has to be substituted in the instability criterion $a > v^2/4$, leading to

$$a \geqslant \frac{v^2(1+vD)^2}{4} \tag{37}$$

as the condition for the appearance of an absolute instability for larger values of a.

E. Time-dependent random velocity

We start from the case of the time-dependent random velocity

$$\frac{\partial\Psi}{\partial t} = \frac{\partial^2\Psi}{\partial x^2} - v[1+\xi(t)]\frac{\partial\Psi}{dx} + a\Psi, \tag{38}$$

where the random force $\xi(t)$ is a Gaussian variable with zero mean and white noise correlator

$$\langle\xi(t)\xi(t_1)\rangle = D_1\delta(t-t_1). \tag{39}$$

After performing the Fourier transform

$$\Psi(x,t) = \int_{-\infty}^{\infty}\tilde{\Psi}(k,t)\exp(ikx)dk, \tag{40}$$

Eq. (38) takes the form

$$\frac{\partial\tilde{\Psi}}{\partial t} = [a - k^2 - ikv - ikv\xi(t)]\tilde{\Psi}. \tag{41}$$

The solution of Eq. (41) with the initial condition $\tilde{\Psi}(t=0) = \tilde{\Psi}_0$ is

$$\tilde{\Psi}(k,t) = \tilde{\Psi}_0\exp[(a-k^2-ikv)t]\left\langle\exp\left(-ikv\int_0^t\xi(\tau)d\tau\right)\right\rangle, \tag{42}$$

which, after using the well-known result $\langle\exp(-ikv)\int_0^t\xi(\tau)d\tau\rangle = \exp[-(k^2v^2D_1/2)t]$ and performing the inverse Laplace transform, gives

$$\Psi(x,t) \approx \exp\left[\left(a - \frac{v^2}{4(1+v^2D_1/2)}\right)t + \frac{2xv - x^2/t}{4(1+v^2D_1/2)}\right]. \tag{43}$$

It follows from Eq. (43) that an instability occurs when

$$a > \frac{v^2}{4(1+v^2D_1/2)}, \tag{44}$$

which is the simple generalization of the condition $a > v^2/4$ in the absence of noise.

F. Time-dependent periodic damping

Let us compare now the results obtained in the previous section with the periodically varying damping described by the equation

$$\frac{\partial\Psi}{\partial t} = \frac{\partial^2\Psi}{\partial x^2} - v[1 + b\cos(\Omega t)]\frac{\partial\Psi}{dx} + a\Psi. \tag{45}$$

On performing calculations similar to Eqs. (38)–(43) one obtains the following solution of Eq. (45):

$$\Psi(x,t) \approx \exp\left[\left(a - \frac{v^2}{4}\right)t + \frac{v[x-(bv/\Omega)\sin(\Omega t)]}{2} - \frac{[x-(bv/\Omega)\sin(\Omega t)]^2}{4t}\right]. \tag{46}$$

The instability occurs when the condition $a > v^2/4$ is satisfied, i.e., the addition of a time-periodic damping does not change the stability condition of the original equation (6).

III. APPROXIMATE SOLUTIONS OF NONLINEAR EQUATION FOR FAST VARYING SPACE-PERIODIC VELOCITY

So far, we have considered the linear version (6) of the non-linear equation (2), which was sufficient for the stability analysis. Note that for the subcritical Ginzburg-Landau equation which contains the third and fifth powers of the order parameter, one has to consider nonlinear stability analysis [18].

Let us consider the parametric force $b\cos lx$ as rapidly oscillating in x. Then, one can consider the nonlinear equation (2) with v replaced by $v(1+b\cos lx)$,

$$\frac{d^2\Psi}{dx^2} - v(1+b\cos lx)\frac{d\Psi}{dx} + a\Psi - b\Psi^3 = 0. \quad (47)$$

Here, one can use an analytical method of separation between the slow scale x and the fast scale lx, similar to the one used for a pendulum with fast oscillation of its suspension point [19]. Let us decompose the function Ψ into a sum of slowly and rapidly varying parts $\Psi=\Psi_1(x)+\Psi_2(x,lx)$, which will be chosen in the following form:

$$\Psi = \Psi_1(x)\left(1 + \frac{vb}{l^2}\cos lx\right). \quad (48)$$

On substituting Eq. (48) into Eq. (47) one obtains two groups of terms, which are either slow ones varying significantly only over distances of the order of x, or fast ones changing over the distances π/l. Performing the averages over distances of order of l^{-1}, one can replace the function $\Psi_1(x)$ by its average over a single cycle length, $\Psi_1(x)\to\overline{\Psi_1(x)}$, while $\overline{\sin lx}$, $\overline{\cos lx}$, and $\overline{\sin^3 lx}$ vanish, $\overline{\sin^2 lx}=\frac{1}{2}$, and one gets finally

$$\frac{d^2\overline{\Psi}}{dx^2} - \left(v + \frac{v^2 b}{2l^2}\right)\frac{d\overline{\Psi}}{dx} + \left(a - \frac{3v^2}{2l^4}\right)\overline{\Psi} - b\overline{\Psi}^3 = 0. \quad (49)$$

Hence, the existence of fast space oscillations of the convective velocity results in small corrections (proportional to small factors $1/l^2$ and $1/l^4$) in the original equation.

IV. STABILITY CONDITIONS FOR A SAMPLE OF FINITE SIZE

The finite size of a sample plays an important role in the interpretation of real experiments. As a matter of fact, for an infinite system the convective term can be simply transformed away by going to a moving frame. Returning now to the original linearized equation (6) one can write the solution of this equation on the finite interval $[0,L]$ with the boundary conditions $\Psi=0$ at $x=0$ and $\Psi=\Psi_0\neq 0$ at $x=L$ in the form [20]

$$\Psi(x,t) = F_1(x) + F_2(x)\exp\left[\left(a - \frac{v^2}{4} - \frac{n^2\pi^2}{L^2}\right)t\right], \quad (50)$$

where $F_1(x)$ and $F_2(x)$ are some functions of x, and $n=1,2,\ldots$. Since the most rapidly growing solution corresponds to $n=1$, a system is absolutely unstable for

$$a \geqslant \frac{v^2}{4} + \frac{\pi^2}{L^2}, \quad (51)$$

i.e., for large L the finite size of a sample results in small change of the condition $a\geqslant v^2/4$.

V. CONCLUSIONS

Recently performed experiments dealing with the propagation of vortices in the presence of a bias current [11] open up a new area for both the experimenter and the theoretician. From the theoretical point of view, the ordered-disordered phenomenon in vortex matter provides another example of phase transitions in moving systems. From the experimental point of view, this branch of research has opened up a chapter of studying the properties and possible new applications of superconducting films. In addition to the dc biased current used in [11], one can use an ac current, or one can use films with periodically ordered pinning centers, which will introduce an additional periodic (in time and space) component to the convective velocity.

Looking forward and trying to trigger these and similar experiments, we considered the theoretical basis for their interpretation. A sharp interface between ordered and disordered moving vortex phases is a quantity immediately measurable by magneto-optical measurements of high temporal resolution [11]. In order to find the stability conditions of a disordered phase for the supercritical Ginzburg-Landau free energy, it is sufficient to perform a linear stability analysis. (This is in contrast to the subcritical case, where the nonlinearity is destabilizing and a nonlinear analysis is required [18].) The stability criteria are formulated in term of the coefficients a and v in our equations, which are proportional to the magnetic field and the bias current, respectively, and by changing these parameters one can go from one regime to another. In the case of a constant dc current, the well-known inequalities $a<0$, $0<a<v^2/4$, and $a>v^2/4$ define the stable, convective unstable, and absolutely unstable regimes, respectively. It turns out that for an additional convective velocity periodic in time these criteria remain unchanged. For an additional space-periodic component one can point out in the amplitude-wave vector the curve (20) which divides the stable and unstable regions.

Noise is a factor which is inherent in all experiments. It is particularly remarkable that the component of the convective velocity random in space results in an increase of stability [see Eq. (37)] while the one random in time decreases the stability [see Eq. (44)].

Additional progress can be achieved when one has a small parameter in the problem. This latter can be a slowly changing or fast varying space-periodic velocity. Seeking the solutions as a series in these parameters, one finds that in both cases the system becomes more stable. The final comment refers to the finite size of the sample in all real experiments. It turns out [see Eq. (51)] that this results in only a small change of the stability criterion.

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